Research Statement

My work focuses on applications of structural graph theory to network games, social networks, internet traffic, and bioinformatics. Complex network theory innovations are a driving force of applications and research in both computer science and mathematics [1, 3, 16]. Structural graph theory began with Euler in the 18th century. Recent work in graph theory, such as the Strong Perfect Graph Theorem by Chudnovsky, continues to be a prolific and vital component of leading combinatorics research [9, 8]. My research combines two distinct components: the theoretical understanding of the structure of graphs, and the applications in which it is being used.

In my doctoral thesis, I defined two complex network generation algorithms by implementing social network theory to lead the use of the deterministic laws for generation [4, 6]. During my first postdoc, I studied eternal-domination, a dynamic graph process that relies heavily on graph structure to determine results which are related to the Spy Game and other computer science applications [11, 12]. In a preprint, I use structural graph theory to determine a forbidden minor result in the game of Cops and Robbers [20, 22, 24, 27]. Finally, in my current postdoctoral position, I leverage the structure of hypergraph networks applied to the biological process of RNA modelling and structure to develop a more advanced understanding of RNA structure [21, 28, 25].

Career Goal: Utilize advanced structural graph theory results and extremal graph theory results in applied settings, efficiently bridging the gap between applied and theoretical combinatorics. Continuing my pursuit of deep structural graph theory results, I will publish papers in games on graphs and network structure algorithms. I will also continue to collaborate with applied mathematicians and computer scientists. To these collaborations, I will bring my experience with deep theoretical results, combining them with applied data and empirical results, deepening our understanding of the world around us, and highlighting the utility and power of mathematics.

Five year goal: Study structural combinatorics in game and algorithmic settings, specifically targeting projects in games on graphs and complex networks that are vulnerable to attack by utilizing structural results such as coloring, saturation and extremal number of graphs, list colourings, hypergraph models, and hypergraph saturation. By building a body of work of structural combinatorics in iterative settings, I will network and collaborate to apply these results to biology, computer programming, environmental science, and other disciplines.

Dissertation Summary and Impact

The *Iterated Local Model (ILM)* is a graph process where at each time-step, either transitive or anti-transitive properties are considered [4]. The ILM is defined in the following way as illustrated in Figure 1 [4]. Given as input a graph G_0 and an infinite binary sequence $S = \{b_0, b_1, ...\}$, at time step t we add a clone x' of each node $x \in V(G_{t-1})$. If $b_t = 1$ we take a transitive step and say the clone x' is adjacent to x and all its neighbours. We call the set of all neighbours the open neighbourhood of x and denote it $N(x)$, and the closed neighbourhood is denoted $N[x]$ is the nieghbourhood including x itself, $N(x) \cup \{x\}$. If $b_t = 0$ we take an anti-transitive step and say the clone x' is adjacent to everything except x and its neighbours. Formally, in time step t we add x' for each $x \in V(G_t)$ with

$$N_{G_t}(x') = \begin{cases} N_{G_{t-1}}[x] & b_t = 1 \\ V(G_{t-1})\backslash N_{G_{t-1}}[x] & b_t = 0 \end{cases}$$

The second model I defined is the Iterated Global Model, where at each time step, vertices are added adjoining to subsets of vertices. In particular, this model generates new vertices based on subsets of the previous vertex set, where the size of the subset is fractional with respect to the size of the vertex set [6]. We begin with a graph G_0 and a parameter $k_0 = k_0(n_0)$ where $n_0 = |V(G_0)|$.

At each time step we create G_{t+1} from G_t by the following algorithm: For each set of vertices of size k_t, add a new v_S that dominates the set. In this particular model, we use the same function of n_t for each time step, although future work could consider a function of both n_t and t. That is, it would be possible to define a sequence of functions that define the $k_t = k_t(n_t)$ for each t. The current research focuses on two models: $k_t = \lfloor \frac{1}{k} n_t \rfloor$ and the $k_t = c$ for some constant c model.

The work in my doctorate provided the expertise and foundation for me to continue my research into complex network models. Recently, I defined and analysed the PALU model, Preferential Attachment, Leaves, and Unattached Links Model [13]. New data was found using trunk-line internet traffic measurements that measure all traffic that passes through some gateway, as opposed to the traditional web-crawl. This data demonstrated the existence of unattached links and leaves in addition to a preferential attachment core. Until this paper, the leading model for internet modelling was Preferential Attachment, with a power law degree distribution [13]. I developed a probabilistic model that generates the existence of these additional components of the Internet and that witnesses a Zipf-Mandlebrot distribution [13]. There are three main pieces that make up this network: the *core* which is constructed by preferential attachment; a set of degree 1 nodes called *leaves* that are adjacent to nodes in the core; and *unattached nodes* that are not connected to the core, and have very low connection within the set itself. These occur in the underlying "true" image of the internet. In the model itself, for each edge in the underlying network, delete it with probability p. In creating this two stage probabilistic network model, we are able to accurately model the sampling techniques to more accurately determine the underlying structure of internet traffic.

Current Research and Further Work

Applying graph structure and model generation to biological applications, I am working with bioinformaticists to further the study of RNA structure. Deterministic generative graph models have been used to study protein-protein interaction networks, social networks, and internet network dynamics [4, 10, 16, 15]. Graph models have worked well in settings limited to two-party interactions but fall short when interactions occur between three or more entities, such as RNA, where hypergraph models are necessary. Towards this end, I first extended work already done in the graph cases to define a deterministic model in this regime [25]. These deterministic models also fall short in their application for modelling real-world scenarios that often include a level of probability in their occurrences. Probabilistic graph models and the Preferential Attachment model can then be modified and superimposed on the hypegraph deterministic model previously defined. This result is an analog of the underlying network and observed network of the PALU model [1, 3, 10, 17, 13]. These new probabilistic hypergraphs will specifically model biological processes, and their structure will then be analyzed [18, 19]. This significant advancement will bring much of the known understanding from graph models into the context of hypergraphs with biological processes as the guiding framework [1, 2, 10, 16, 18, 26].

From the structural graph theory side of my research, I currently apply my knowledge to games on graphs. I have publications in three distinct networks games: Cops and Robbers, Explorer-Director, and Eternal Domination [22, 12, 14]. Graph searching is the study and analysis of graph processes whose aim is to eliminate some sort of intrusion in a network, and is a key component in both Cops and Robbers and Eternal Domination [22, 12]. The game of Cops and Robbers is an extensively researched game where there is a set of cops that attempts to capture a robber loose on the network each moving from vertex to vertex along the edges on their respective terms [24, 22, 7, 23, 5]. For a graph G, the cop number, denoted $c(G)$, is the minimum number of cops necessary where the cop player has a winning strategy.

In my masters thesis, I defined the game *Cops, and Robbers, and Barricades (CRB)*, which is played on a reflexive graph G. Analogous to the original game, the cops and robbers move the same way with an additional rule, the robber, in round k, is allowed to move to any adjacent vertex or instead may build $b(k)$ many barricades. The barricade cop number of a graph, denoted $c_B(G)$, is the number of cops needed to guarantee a win for the cops [22], and in this thesis I determined this number for a variety of classes of graphs. In addition, I determined the complexity of the problem.

In my current coauthored project, I investigate the cop number in terms of higher dimensional graph embeddings, namely linkless embeddings. A linkless embedding of a graph is an embedding of the graph into $\mathbb{R}^3$ such that every pair two disjoint cycles forms a trivial link, that is they do not pass through each other [**?**]. In this work, I have discovered a forbidden minor result, where we can bound the cop number given that there does not exist a minor of a certain structure. This research is ongoing, as I continue to determine other consequences of these structural results.

Mentorship

In my postdoctoral experience, I have a strong track record of publishing with my supervised students. In Summer 2020, I was a research co-mentor for the Polymath REU, a research experience for around 60 undergraduate students from all over the world, where we studied games on graphs. The project on the Explorer-Director game lead to a submitted publication [14]. Many of these students were exceptional researchers that I wrote reference letters for in 2020. In Fall 2020, I worked closely with an undergraduate student at MIT, Ashley Luo, on the PALU Model project paper [13]. Her work on the project provided insight to the probabilistic analysis.

In Spring 2019, I mentored Saje Bailey at Wilfrid Laurier University during the thesis course CP493, where we studied applications of network games in cryptography. This project aimed to create a cryptosystem using the complexity of the game Cops, Robbers, and Barricades, which is in EXPTIME unless the barricade sequence is known, in which case it is in P [22]. Saje would be interested in continuing this research in a Master's degree under my supervision if I am in a position to do so when they apply for graduate school after the pandemic.

I am very interested to work with students on games on graphs and iterated generative models. These problems can arise from a variety of applications, and allow students to create their own variants according to their interests. In this area, there is a typical structure for research as accepted by the community: determine results for simple graph classes, then more complicated graph classes, and finally analyzing graph parameters or complexity. Students can create their own research plan under supervision, while exploring and understanding a broad selection of literature. This provides an avenue for creative research as well as deepening the understanding of current graph theory research. Individuals from traditionally underrepresented groups in STEM are lost not due to lack of interest or aptitude, but due to the leaky pipeline that is designed *not* to support them. The topics I have worked on with students were intentionally selected to invite participation from researchers with all kinds of backgrounds with a view to create inclusive, productive, and rigorous collaborations. Graph Theory can only be enhances by making research more accessible to a wider group of participants. In light of the dire need for proper representation of diversity within mathematical researchers, I am particularly commited to mentoring students who are women, LGBTQ+, or BIPOC.

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