SC101 Exam Review

1 Functions and Arithmetic

1.1 Arithmetic

1. Evaluate:

   (a) $\left(\frac{1}{4}-\frac{1}{5}\right)^{-1}$

   (b) $\left(\frac{1}{2}-1\right)^{-2}$

   (c) $\left(\frac{2}{5}-\frac{1}{2}+1\right)^{-2}$

   (d) $\left(\frac{3}{4}+\frac{7}{6}-\frac{1}{3}\right)^{-1}$

2. Simplify:

   (a) $\sqrt{3}+\sqrt{12}-\sqrt{27}$

   (b) $(\sqrt{5}+3\sqrt{5})^{-2}$

   (c) $\left(\sqrt{3}-4\sqrt{3}\right)^{-1}$

   (d) $\left(3\sqrt{2}-4\sqrt{2}\right)^{2}$

3. Factor the following fully:

   (a) $9x^2-64$

   (b) $(3x)^2-49$

   (c) $x^2+3x-10$

   (d) x^2+x-42

   (e) $2x^2-x+6$

   (f) $\frac{x^2}{4}-16$

4. Solve:

   (a) $\frac{3-2x}{4}=7x+1$

   (b) $4t\sqrt{2}+7=-1$

   (c) $|x+1|=10$

   (d) $|7x-2|+1=0$

   (e) $y^2+4y-1=0$

   (f) $10=7x-x^2$

   (g) $\frac{3x+3}{x-3}=7$

   (h) $\frac{2x}{x-3}=4+\frac{6}{x-3}$

   (i) $\frac{3x-1}{x-1}=x+1$

5. Solve the following and write your answer in interval notation:

   (a) $\dfrac{3+x}{4} \geq 1 + 2x$

   (b) $5 - (2 - x) < 2x + 1$

   (c) $|x| \geq 2$

   (d) $|x| < 3$

   (e) $|x + 1| \leq 2$

   (f) $|x| - 1 > 1$

6. Simplify the following rational expressions:

   (a) $\dfrac{1}{x+1} + \dfrac{2}{x-3}$

   (b) $\dfrac{x}{x+1} - \dfrac{1}{x-3}$

   (c) $\dfrac{x+1}{x-1} + \dfrac{2x-1}{x+5}$

   (d) $\dfrac{2x-1}{x+3} - (x+4)^{-1}$

1.2 Function Arithmetic, Composition, and Inverses

7. Find the domain of each of the given functions f, g, and h. Then, determine the following:

   (i) $\left(\dfrac{f+h}{g}\right)(x)$

   (ii) $(f - h)(x)$

   (iii) $(f \circ h \circ g)(x)$

   (iv) $h(g(x))$

   (v) $f((h+g)(x))$

   (vi) $g((f-h)(x))$

   (a)
   - $f(x) = \sqrt{x}$
   - $g(x) = x^2 - 1$
   - $h(x) = x + 1$

   (b)
   - $f(x) = \sin(x)$
   - $g(x) = 3x + 1$
   - $h(x) = \frac{1}{2}x^2$

   (c)
   - $f(x) = \cos(x)$
   - $g(x) = \sqrt{x+1}$
   - $h(x) = x - 1$

8. For each of the given functions, find three functions $f(x), g(x)$, and $h(x)$ such that $F(x) = f(g(h(x)))$:

   (a) $F(x) = 3\left(\sin\left(\sqrt{x}\right)\right)^2 - 1$

   (b) $F(x) = \dfrac{\ln(x)}{2x^2 - 1}$

   (c) $F(x) = \dfrac{\left(\sqrt{x}+1\right)^2}{2\left(\sqrt{x}+1\right)^3 - 1}$

9. Find the inverse function $f^{-1}(x)$ for each of the following invertible functions:

(a) $y = \dfrac{x+1}{3x-1}$

(b) $y = \dfrac{2x-1}{5x+1}$

(c) $y = \dfrac{2x}{1-5x}$

(d) $y = \dfrac{3x-1}{2x+5}$

(e) $y = \frac{1}{2}\sqrt{x-1}+2$

(f) $y = 3\sqrt{2x+9}-1$

1.3 Functions

10. Find the equation of the line through each of the following pairs of points. Write your final answer in both point-slope form and slope-intercept form.

(a) $\left(\sqrt{2},3\right);\left(-2\sqrt{2},0\right)$

(b) $\left(1,2\sqrt{3}\right);\left(\sqrt{3},-1\right)$

(c) $\left(\dfrac{2}{3},\dfrac{1}{2}\right);\left(\dfrac{4}{3},\dfrac{1}{5}\right)$

(d) $\left(\dfrac{3}{4},1\right);\left(\dfrac{1}{2},2\right)$

11. Evaluate and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for each:

(a) $f(x) = 3x^2+1$

(b) $f(x) = x^2-9$

(c) $f(x) = 2x+5$

(d) $f(x) = \dfrac{1}{2x}$

12. Determine if the following functions are even, odd or neither.

(a) $f(x) = x^2-1$

(b) $f(x) = \dfrac{1}{x^2+1}$

(c) $f(x) = \sqrt{x+1}$

(d) $f(x) = \dfrac{1}{\sqrt{x-1}}$

(e) $f(x) = 2^x$

(f) $f(x) = \sqrt{x^2-1}$

(g) $f(x) = \sin(x)$

(h) $f(x) = \cos(x)$

13. Using each graph of $y = f(x)$ given below, graph the following transformations:

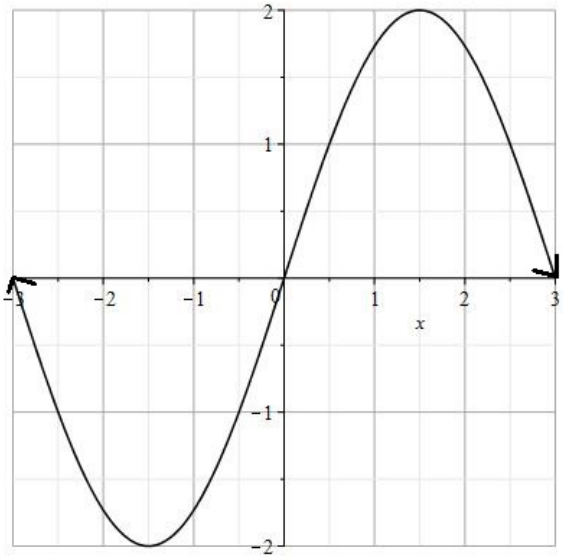
(i) $f(x-3)+1$

(ii) $f(x+1)$

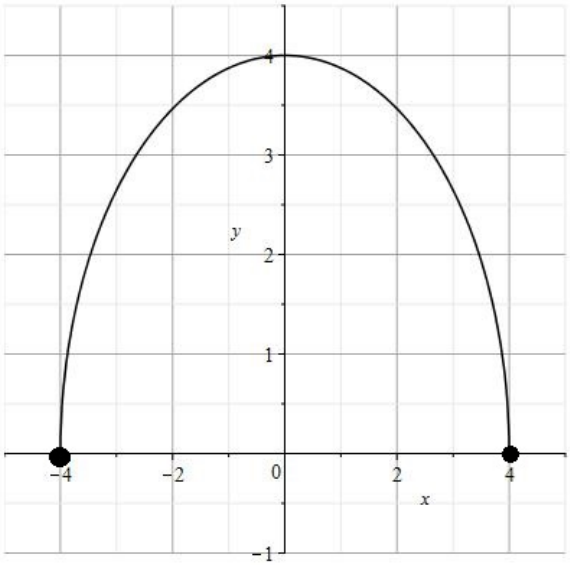
(iii) $f(x)+4$

(iv) $2f(x-1)+1$

(a)



(b)



14. Pooja and Christian opened an espresso shop on campus. They are only open from 10:00 p.m.- 3:00 a.m. to capitalize on student exam stress. They sell regular espresso for 75 cents per shot, they sell drip coffee for $1.50, and they sell pumpkin spice lattes for $3.75. They sell twice as many espresso shots as they do orders of drip coffee, and they sell three times as many drip coffees as pumpkin spice lattes. This week they made $229.50. How many of each type of beverage did they sell?

15. Your friend at Western University plans to visit you next semester during St. Patrick's Day weekend. To get to Waterloo, she needs to rent a car. There are two possible rental companies: UWO Cars charges a flat rate of $120 plus 25 cents per kilometre. London Rentals charges $175 for the first 50 kilometres, plus 15 cents for each additional kilometre (for example, a 52 km trip costs $175 plus 30 cents). What trip distance (in kilometres) makes the cost of each plan equal?

16. Driving east along Highway 8, Hannah travels $x - 17$ kilometers. Then, she takes the Highway 6 exit and travels north for $x + 1$ kilometers. Hannah suddenly realizes she's forgotten all about her math test and begins driving back to Waterloo, turning left on Line 85 and then left again onto Northfield Drive (eventually returning to her starting point). Although none of the streets in Waterloo Region run in any semblance of a grid, assume this forms a perfect rectangle. If Hannah travels 84 km in total, what is the area of the rectangle formed by her journey?

17. A frustrated SC101 student throws her CASIO FX-300MS Plus calculator from her bedroom window on the eighth floor of her residence building. The height of the calculator above the ground, in metres, t seconds after being thrown, is modeled by

$$h(t) = -5t^2 + 10t + 15$$

    (a) When will the calculator hit the ground?

    (b) What is the maximum height of the calculator?

18. A dolphin dives in the ocean. Her depth below the water can be modeled by

$$d(t) = 6t^2 - 7t + 2$$

    (a) When will the dolphin reach the surface?

    (b) What is the minimum depth of the dolphin?

19. π Pizza Company charges \$9.99 for one medium pizza, plus a \$3.14 delivery charge. They offer a "Toonie Tuesday" Special: each medium pizza purchased after the first two costs \$2.00 (delivery is still a \$3.14 flat rate). Write a piecewise-defined function which calculates the cost C in dollars of p medium pizzas delivered on a Tuesday.

2 Trigonometry

20. Convert the following angles from degrees to radians.

    (a) $210°$ (b) $300°$ (c) $-315°$ (d) $-36°$

21. Convert the following angles from radians to degrees.

    (a) 4π (b) $-\dfrac{7\pi}{2}$ (c) 5 (d) $\dfrac{5\pi}{12}$

22. Evaluate the following to one decimal place.

    (a) $\sin(250°)$ (b) $\csc\left(\dfrac{\pi}{7}\right)$ (c) $\tan(12)$ (d) $\sec(-16°)$

23. Draw the following angles in standard position and find each of the six trigonometric ratios.

    (a) $\dfrac{-11\pi}{4}$ (d) $\dfrac{10\pi}{3}$ (g) $\dfrac{8\pi}{2}$ (j) $\dfrac{5\pi}{3}$

    (b) $\dfrac{13\pi}{6}$ (e) $\dfrac{-5\pi}{4}$ (h) $\dfrac{-14\pi}{3}$ (k) $\dfrac{-7\pi}{2}$

    (c) $\dfrac{-7\pi}{2}$ (f) $\dfrac{5\pi}{6}$ (i) $\dfrac{13\pi}{6}$ (l) $\dfrac{7\pi}{3}$

24. Use the given information to find the exact values of the remaining trigonometric ratios of θ.

    (a) $\sin(\theta) = -\frac{3}{5}$ with $\pi < \theta < \dfrac{3\pi}{2}$.

    (b) $\sec(\theta) = 4$ with θ in Quadrant II.

    (c) $\cot(\theta) = 2$ with θ in Quadrant I.

    (d) $\csc(\theta) = \frac{25}{24}$ with $0 < \theta < \dfrac{\pi}{2}$.

    (e) $\sec(\theta) = 7$ with θ in Quadrant IV.

25. Find all of the angles θ, with $0 \leq \theta \leq 2\pi$, which satisfy the following equations:

    (a) $\sin(\theta) = -\frac{1}{\sqrt{2}}$
    (b) $\tan(\theta) = -\sqrt{3}$
    (c) $\sin(\theta) = 0$
    (d) $\cos(\theta) = -1$
    (e) $\sin(\theta) = -\frac{\sqrt{3}}{2}$
    (f) $\cos(\theta) = \frac{\sqrt{3}}{2}$
    (g) $\csc(\theta) = \frac{2}{\sqrt{3}}$
    (h) $\sec(\theta) = -\sqrt{2}$
    (i) $\cot(\theta) = 1$
    (j) $\csc(\theta) = 2$

26. For each of the following trigonometric functions, state the key features (i.e., the amplitude, period, phase shift, and vertical shift.).

    (a) $\frac{1}{3}\sin\left(\pi\left(x - \frac{\pi}{4}\right)\right) + 2$
    (b) $2\sin\left(\frac{\pi}{2}x - \frac{1}{3}\right) - 1$
    (c) $\frac{2}{3}\cos\left(3x - \frac{\pi}{2}\right) + \frac{\pi}{4}$
    (d) $3\cos\left(\frac{1}{2}\left(x - \frac{\pi}{6}\right)\right) + \frac{\pi}{4}$

3 Exponential and Logarithmic Functions

27. Rewrite the following in exponential form:

    (a) $\log_3\left(\frac{1}{27}\right) = -3$
    (b) $\log_4(64) = 3$
    (c) $-9 = \log_2\left(\frac{1}{512}\right)$
    (d) $4 = \log_5(625)$

28. Rewrite the following in logarithmic form:

    (a) $\frac{1}{16} = 2^{-4}$
    (b) $3^{-2} = \frac{1}{9}$
    (c) $256 = 4^4$
    (d) $5^3 = 125$
    (e) $6^1 = 6$
    (f) $1 = \pi^0$

29. Evaluate the following exactly:

(a) $\log_{15}(25) + \log_{15}(9)$

(b) $\ln(\sqrt{e}) + \ln\left(\frac{1}{e}\right)$

(c) $\log_{12}(36) + \log_{12}(48)$

(d) $\ln\left(\frac{1}{\sqrt{e}}\right)$

(e) $\log_{15}\left(\frac{1}{25}\right) + \log_{15}(9)$

(f) $\log_6(648) - \log_6(3)$

30. Use the properties of logarithms to expand and simplify the following expressions:

(a) $\log_2\left(\dfrac{8\sqrt{xy}}{y^{-2}}\right)$

(b) $\ln\left(\dfrac{x^2}{(xy)^3}\right)$

(c) $\log_3\left(\dfrac{x^2}{81y^4}\right)$

(d) $\ln\left(\sqrt{e^3y^4}\right)$

31. Write the following as a single logarithm:

(a) $3\ln x - 2\ln y + \frac{1}{2}\ln z$

(b) $3\log_2 xy^2 - \log_2 zx$

(c) $2\log_3 x + 3\log_3 y - 3\log_3 xy + \frac{1}{2}\log_3 x$

(d) $\frac{1}{2}\ln x - \frac{1}{3}\ln y + \frac{3}{4}\ln z$

32. Solve the following exponential equations for x.

(a) $4^{2x} = \frac{1}{2}$

(b) $2^{4x} = 8$

(c) $3^{7x} = 81^{4-2x}$

(d) $3^{x-1} = 27$

(e) $e^{2x} = 2e^x$

(f) $7^{3+7x} = 49^{2-x}$

(g) $8^{x^2} = 2^{2x+5}$

(h) $e^{2x} = e^x + 6$

(i) $3^x + 25\,(3^{-x}) = 10$

33. Solve the following logarithmic equations for x.

(a) $\ln(8 - x^2) = \ln(2 - x)$

(b) $\ln(x^2 - 99)$

(c) $\log_{125}\left(\dfrac{3x-2}{2x+3}\right) = \dfrac{1}{3}$

(d) $\log_3(x-4) + \log_3(x+4) = 2$

(e) $2\log_7(x) = \log_7(2) + \log_7(x+12)$

(f) $\log(x) - \log(2) = \log(x+8) - \log(x+2)$

34. Solve the following textbook problems:

(a) §6.1 # 75ab
(b) §6.1 # 76
(c) §6.5 # 9
(d) §6.5 # 22
(e) §6.5 # 24
(f) §6.5 # 27
(g) §6.5 # 31