Odd Cycle Saturation Games

Dr. Erin Meger

Ryerson University
Graduate Research Workshop in Combinatorics
Joint work with S. English, T. Masařík, G. McCourt M. Ross S. Spiro

JMM 2020
January 18

Table of Contents

Sean

Tomás Masařík

Grace McCourt

Mike Ross

Sam Spiro

Me

The Team


PRACTICE EXAMS MAKE PERFECT EXAMS
Graduate Research Workshop in Combinatorics

Working hard

Working hard

Saturation

Definition

A graph G is said to be $\mathcal{F}$-free if no subgraph of G is isomorphic to any graph $F \in \mathcal{F}$.

Saturation

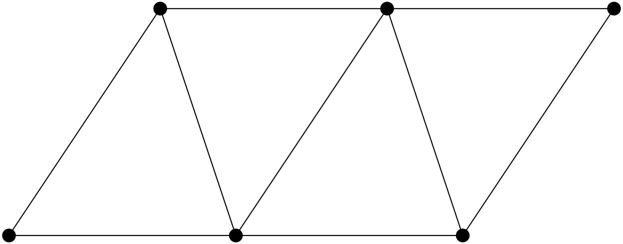
Definition

A graph G is said to be $\mathcal{F}$-free if no subgraph of G is isomorphic to any graph $F \in \mathcal{F}$.

Definition

A graph G is said to be $\mathcal{F}$-saturated if G is $\mathcal{F}$-free and for any edge $e \notin E(G)$ then $G + e \in \mathcal{F}$

Example



A Graph that is K_4-free but not K_4-saturated

Saturation Number

Definition

The saturation number of $\mathcal{F}$ is the **minimum** number of edges in an $\mathcal{F}$-saturated graph with n vertices, denoted

$$\text{sat}(n, \mathcal{F})$$

Saturation Number

Definition

The saturation number of $\mathcal{F}$ is the **minimum** number of edges in an $\mathcal{F}$-saturated graph with n vertices, denoted

$$\mathrm{sat}(n, \mathcal{F})$$

Definition

The extremal number of $\mathcal{F}$ is the maximum number of edges in an $\mathcal{F}$-free graph with n vertices, denoted

$$\mathrm{ex}(n, \mathcal{F})$$

Hajnal's Original Game

- Two Players take turns adding an edge to an empty graph
- They cannot add an edge that forms a triangle
- The last player to add an edge wins

Hajnal's Original Game

- Two Players take turns adding an edge to an empty graph
- They cannot add an edge that forms a triangle
- The last player to add an edge wins

Question: Who wins as a function of n?

Hajnal's Original Game

- Two Players take turns adding an edge to an empty graph
- They cannot add an edge that forms a triangle
- The last player to add an edge wins

Question: Who wins as a function of n?

For $3 \leq n \leq 9$ player 2 wins except if $n = 6$.

Füredi, Reimer, and Seress Game

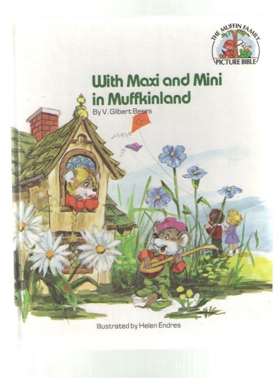
Two Players: Mini and Maxi
Final Score:

- Mini: -1 point for every edge
- Maxi: +1 point for every edge

Füredi, Reimer, and Seress Game

Two Players: Mini and Maxi
Final Score:

- Mini: -1 point for every edge
- Maxi: +1 point for every edge



Saturation Games

Definition

The game saturation number of a graph family $\mathcal{F}$ is the number of edges in the resulting graph from the saturation game avoiding $\mathcal{F}$ with Mini and Maxi playing optimally

$$\mathrm{sat}_g(\mathcal{F}; n)$$

Saturation Games

Definition

The game saturation number of a graph family $\mathcal{F}$ is the number of edges in the resulting graph from the saturation game avoiding $\mathcal{F}$ with Mini and Maxi playing optimally

$$\mathrm{sat}_g(\mathcal{F}; n)$$

Theorem

$$\frac{1}{2} n \log n + o(n \log n) \leq \mathrm{sat}_g(C_3; n)$$

Saturation Games

Definition

The game saturation number of a graph family $\mathcal{F}$ is the number of edges in the resulting graph from the saturation game avoiding $\mathcal{F}$ with Mini and Maxi playing optimally

$$\text{sat}_g(\mathcal{F}; n)$$

Theorem

$$\frac{1}{2} n \log n + o(n \log n) \leq \text{sat}_g(C_3; n)$$

Erdős

$$\text{sat}_g(C_3; n) \leq \frac{n^2}{5}$$


Allegedly